Test Bank Exercises in

CHAPTER 3

Exercise Set 3.1

1. Use the midpoint and the distance formulas, respectively, to find (a) the midpoint and (b) the distance between the points (6, 5) and (–1, 4).

2. Use the midpoint and the distance formulas, respectively, to find (a) the midpoint and (b) the distance between the points (1/2, –3) and (1, 0).

3. Use the midpoint and the distance formulas, respectively, to find (a) the midpoint and (b) the distance between the points ($\sqrt{3}$, 5/3) and (0, 1/3).

4. Given the points $A = (-4, 3)$, $B = (-5, 7)$, and $C = (-1, 6)$, use the distance formula to find the lengths *AB*, *BC*, and *CA*, and determine whether the triangle *ABC* is (a) a right triangle, (b) an isosceles, (c) an equilateral triangle, or (d) neither.

5. Give the points $A = (1, 1)$, $B = (-2, 4)$, and $C = (3, 3)$, use the distance formula to find the lengths *AB*, *BC*, and *CA*, and determine whether the triangle *ABC* is (a) a right triangle, (b) an isosceles, (c) an equilateral triangle, or (d) neither.

6. Given the points $A = (-1, 1)$, $B = (6, 3)$, and $C = (1, -6)$, use the distance formula to find the lengths *AB*, *BC*, and *CA*, and determine whether the triangle *ABC* is (a) a right triangle, (b) an isosceles, (c) an equilateral triangle, or (d) neither.

7. Given the points $A = (1, 3)$, $B = (8, 5)$, and $C = (3, -4)$, use the distance formula to find the lengths *AB*, *BC*, and *CA*, and determine whether the triangle *ABC* is (a) a right triangle, (b) an isosceles, (c) an equilateral triangle, or (d) neither.

8. Given the points $A = (-1, 1)$, $B = (3, 1)$, and $C = (1, 1 + 2\sqrt{3})$, use the distance formula to find the lengths *AB*, *BC*, and *CA*, and determine whether the triangle *ABC* is (a) a right triangle, (b) an isosceles, (c) an equilateral triangle, or (d) neither.

9. Using the formula $Area = \frac{1}{2}(base)(height)$, find the area of the right triangle whose vertices are (–3, –1), (4, 1), and (–1, –8).

10. The *Heron's formula* says that if a, b, and c are the side lengths of a triangle, and $2s = a + b + c$, then the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$. Find the area of each triangle in problems 4 to 8 above.

11. The area of a triangle whose vertices are the points (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by $A = \frac{1}{2}|x_1y_2 - x_2y_1 + x_2y_3 - x_3y_2 + x_3y_1 - x_1y_3|$

    Use this formula to find the area of each triangle in problems 4 to 8 above.

12. Three vertices of a square are (4, –1), (0, –3), and (2, 3). Find the fourth vertex.

13. An end point and the midpoint of a line segment are respectively (3, –5) and (–1, 2). Find the other endpoint.

14. Without graphing, determine if the points (–1, 0), (7, 4), and (11, 6) lie on the same straight line.

15. Sketch the parallelogram with vertices (–5, 1), (6, 5), (9, 10), and (–2, 6). Find the midpoints of the two diagonals of this parallelogram. Can you draw any conclusion from this?

16. Find (a) all possible intercepts, (b) all possible symmetries and graph the equation $y = \sqrt{x+1} - 2$.

17. Find (a) all possible intercepts, (b) all possible symmetries and graph the equation $y^2 = x^3$.

18. Find (a) all possible intercepts, (b) all possible symmetries and graph the equation $|x| + |y| = 1$.

19. Find (a) all possible intercepts, (b) all possible symmetries and graph the equation $y = \sqrt{4 - x^2}$.

20. Find (a) all possible intercepts, (b) all possible symmetries and graph the equation $x = \sqrt{4 - y^2}$.

Exercise Set 3.2

In problems 1 to 5 determine whether the given equation determines y as a function of x. In case it does, write the function and find its domain.

1. $2x + 3y = 6$

2. $x - y^2 = 1$

3. $y + 2x^2 = 6$

4. $y = \sqrt{x - 2}$

5. $y^3 - x = 0$

6. Write the function defined by the equation $y = |x - 3| + 1$, and find its domain. Use the graph of this equation to find the range of this function.

7. Given the function $f(x) = x^2 - x + 4$, find the following
   (a) $f(-3)$
   (b) $f(-x)$
   (c) $-f(x)$
   (d) $f(1/x)$
   (e) $\dfrac{f(1 + h) - f(1)}{h}$

8. Given the function $f(x) = \dfrac{x + 6}{x^2 - 4x + 3}$, answer the following
   (a) Find domain of f
   (b) Find $f(1/x)$
   (c) Find $1/f(x)$
   (d) Find $f(1 + h)$

9. An object is thrown vertically up and its height at time t (measured in seconds) is given by the formula
   $$h(t) = -16t^2 + 256t$$
   (a) Find the height of the object at time 4 seconds.
   (b) Find the time when the object hits the ground.
   (c) Find the distance travelled by the object in the fourth second of its motion.

In problems 10 to 15, graph the functions defined by the given rules.

10. $f(x) = \begin{cases} x^2 \text{ if } -3 \leq x \leq 0 \\ \sqrt{x} \text{ if } x > 0 \end{cases}$

11. $f(x) = \begin{cases} x^3 \text{ if } x < 0 \\ x^2 \text{ if } x > 0 \end{cases}$

12. $f(x) = \begin{cases} \sqrt[3]{x} \quad x \leq 0 \\ \sqrt{x} \quad x > 0 \end{cases}$

13. $f(x) = \begin{cases} 1 \text{ if } x < 1 \\ 1/x \text{ if } x \geq 1 \end{cases}$

14. $f(x) = \dfrac{|x|}{x}, x \neq 0$

15. $f(x) = \begin{cases} 2x - 3, \text{ if } -4 < x \leq 3 \\ 1 - 2x \text{ if } 3 < x < 5 \end{cases}$

16. Sketch the graph of the function $f(x) = \dfrac{x^2 - 4}{x + 2}$. What is the domain of f?

17. Sketch the graph of the function $f(x) = 2\sqrt{1 - x^2 / 9}$. Find the domain and the range of f.

18. Let $f(x) = 1/x$. Find $\dfrac{f(x + H) - f(x)}{h}$. What is the domain of f?

19. Let $f(x) = \dfrac{x - 1}{x + 1}$. Find the following

(a) Domain of f.

(b) $f\left(\dfrac{x - 1}{x + 1}\right)$.

(c) f(1/x).

20. Let $f(x) = \dfrac{4}{3}\pi x^3$. Graph the function f and find the following

(a) f(1.2)

(b) $\dfrac{f(1.2) - f(1)}{.2}$

Exercise Set 3.4

In exercises 1 to 8, find an equation of the line satisfying the given conditions. Express your answer in the form $y = mx + b$.

1. Passing through the points (–1, 4) and (3, 5).

2. Passing through the point (1/2, 4) and has slope $m = -3$.

3. Has slope –1/2 and x-intercept 5/3.

4. Has x-intercept –4 and y-intercept 5.

5. Has y-intercept 3/2 and is parallel to the x-axis.

6. Passes through the point (–1, 1) and is parallel to the line $3x + 4y - 7 = 0$.

7. Passes through the point (5, –2) and is perpendicular to the line $x - 2y - 4 = 0$.

8. Has slope 2 and x-intercept 0.

For each of the following pairs of lines given in problems 9 to 11, indicate if the lines are (a) parallel, (b) perpendicular, or (c) neither.

9. $\begin{cases} L_1\colon 2x - 3y + 7 = 0 \\ L_2\colon 4x - 6y + 1 = 0 \end{cases}$

10. $\begin{cases} L_1\colon 4x - y = 0 \\ L_2\colon x + 4y = 0 \end{cases}$

11. $\begin{cases} L_1\colon 2x + 7y - 15 = 0 \\ L_2\colon 2x - 7y - 15 = 0 \end{cases}$

12. Find an equation of the line that crosses the x-axis at the point (–3, 0) and is parallel to the line $5x - 7y = 11$. Graph both lines on the same set of axis.

13. Find an equation of the line that crosses the y-axis at the point (0, 4) and is perpendicular to the line $y = x$. Graph both lines on the same set of axis.

14. For what value of c will the lines $2y - x + 5 = 0$ and $(1 + c)x + y + 2 = 0$ be parallel?

15. For what value of c will the lines $2x + cy - 3 = 0$ and $4x - 9y + 1 = 0$ be perpendicular?

16. Can the three points (–1, 4), (1, 5), and (3, 5) lie on the same straight line? Explain why.

17. Are the lines $y = 3x + 2$ and $y = 2x - 3$ parallel, perpendicular, or neither?

18. Assume that your earnings follow a linear growth pattern given by $E = at + b$, where t is measured in years. In 1989 you earned \$25,000 and in 1991 you earned \$25,000. What will your earnings be in the year 1996? Graph your earnings as a function of time.

19. Assume that your earnings follow a linear growth pattern given by $E = at + b$, where t is measured in years. In 1987 you earned \$21,500 and in 1990 you earned \$28,700. What will your earnings be in the year 1998? Graph your earnings as a function of time.

20. Suppose that the cost C in dollars of producing x number of quantities of a certain product is given by $C = 1350 + 65x$. Graph C as a function of x. What is the marginal cost? (Recall that the marginal cost is the cost to produce one more unit.)

Exercise Set 3.5

In problems (1) to (5) find the following functions and the domains of these functions.

(a) $f + g$; (b) $f - g$; (c) $(3f - 2g)$; (d) fg; (e) $\frac{f}{g}$; (f) $\frac{1}{g}$; (g) $\frac{1}{f}$; (h) $f \circ g$; (i) $g \circ f$; and (j) $f \circ f$

1. $f(x) = 3x + 5, g(x) = \dfrac{x - 5}{3}$

2. $f(x) = \sqrt{3x + 2}, g(x) = x^2 + 1$

3. $f(x) = 5x^2 + 4, g(x) = \dfrac{1}{\sqrt{|x|}}$

4. $f(x) = 2x + 1, g(x) = x^3$

5. $f(x) = x^3 + 1, g(x) = \sqrt[3]{x - 1}$

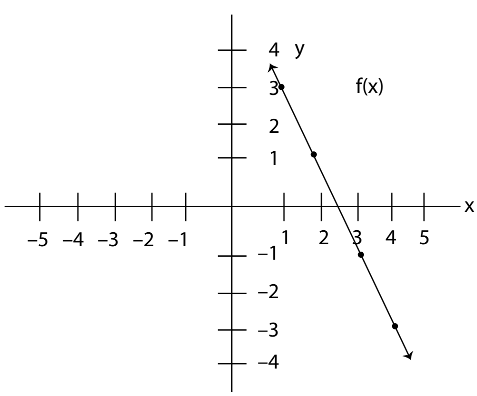
6. Given that $f(x) = \sqrt{x^2 - 1}$ and $g(x) = 2x + 1$, find the following:

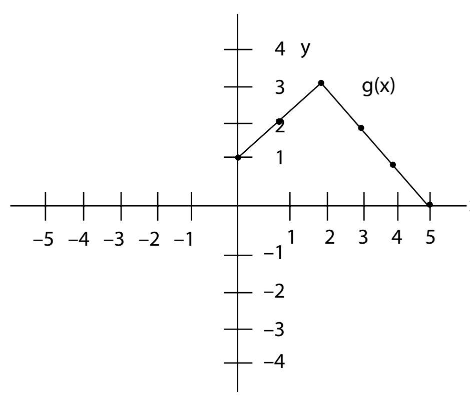
   (a) $(f + g)(2)$ (b) $(fg)(1)$ (c) $f(g(1))$ (d) $g(f(-1))$ (e) $\left(\dfrac{f}{g}\right)(2)$

7. Let $f(x) = 3x + 5$. Find $f(f(x))$.

8. Let $f(x) = x^2 + 1$. Find $f(f(x))$.

In exercises (9) through (14) refer to the graphs of the functions and find the following





9. $(f + g)(1)$

10. $(fg)(4)$

11. $\dfrac{f}{g}(2)$

12. $(f \circ g)(4)$

13. $(f \circ (g \circ f))(2)$

14. $((f \circ g) \circ f)(2)$

15. Given $h(x) = \sqrt{x^2 + 1}$, find the functions f and g such that $f \circ g = h$.

16. Given $h(x) = (3x - 2)^2 + 4$, find the functions f and g such that $f \circ g = h$.

17. Given $h(x) = (x+4)^2 + 3(x+4)$, find the functions f and g such that $f \circ g = h$.

18. Given $h(x) = \dfrac{1}{(2x-5)^3}$, find the functions f and g such that $f \circ g = h$.

19. Find the inverse of the function $f(x) = 4x - 7$.

(a) $f^{-1}(x) = \dfrac{x+7}{4}$ (b) $f^{-1}(x) = \dfrac{x-7}{4}$

(c) $f^{-1}(x) = \dfrac{1}{4x-7}$ (d) None of the above.

20. Find the inverse of the function $f(x) = x^2 + 9$, $x \geq 0$.

(a) $f^{-1}(x) = x^{-2} + 9$ (b) $f^{-1}(x) = \sqrt{x-9}$

(c) $f^{-1}(x) = \dfrac{1}{x^2+9}$ (d) None of the above.

21. Find the inverse of the function $f(x) = x^3 - 1$.

(a) $f^{-1}(x) = \sqrt[3]{x+1}$ (b) $f^{-1}(x) = \sqrt[3]{x} - 1$

(c) $f^{-1}(x) = \dfrac{1}{x^3-1}$ (d) None of the above.

22. Find the inverse of the function $f(x) = 3x - 1$. Graph both functions f and f^{-1} on the same coordinate axis.

23. Find the inverse of the function $f(x) = x^3$. Graph both functions f and f^{-1} on the same coordinate axis.

Exercise Set 3.6

In exercises (1) to (4) translate the given statement into an equation.

1. y varies as the cube root of x.

2. y varies inversely as the square of x.

3. z varies directly as x and inversely as square of y.

4. w varies jointly as u^2 and v^3.

5. If y varies inversely as cube root of x and $y = 1$ when $x = 8$, find y when $x = 27$.

6. z varies jointly as x and y. If $z = 6$ when $x = 2$ and $y = 4$, find z when $x = 3$ and $y = 8$.

7. The area of a circle varies as the square of the radius. If the area A is $9\pi\text{cm}^2$ when the radius r is 3cm, find the constant of proportionality.

8. z varies jointly as the square of x and square root of y and inversely as the cube of w. If $z = 1/2$ when $x = 2$, $y = 9$ and $w = 3$, find z when $x = 1$, $y = 9$ and $w = -2$.

9. Given that an object falls from rest and freely under gravity from a height 512 feet in such a way that its height at time t seconds is proportional to square of the time. If it falls 144 feet in the first 3 seconds, how long will it take to hit the ground?

10. According to the *Boyle's Law,* under constant temperature, the volume V of a gas is inversely proportional to the pressure P. If a certain gas occupies 15.5 liters at a pressure of 75 cm.Hg., find the pressure if the volume is decreased to 10 liters.

11. According to the *Newton's Law of Gravitation,* the gravitational attraction F between two bodies having masses m_1 and m_2, respectively, is proportional to the product of the masses and inversely proportional to the square of the distance r between them. Express this statement as an equation. If $F = 500$ ft.lb. when $m_1 = 275$ lb., $m_2 = 180$ lb., and $r = 275$, find F when $m_1 = 100$, $m_2 = 125$, and $r = 50$.